Coupling Between Narrow Transverse Inductive Strips in Waveguide

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Abstract—**A general expression is found for the susceptance of two narrow transverse strips of differing width, unsymmetrically located in a rectangular waveguide. This analysis is based on extremization of the current-density ratio between the two strips, through use of the variational principle. The resulting susceptance values have been experimentally verified, and the theory is valid for interstrip spacings ranging down to the point where the two strips touch, or even overlap.**

I. INTRODUCTION

THIS paper sets out an analysis of the coupling between two narrow, transverse, inductive strips which are of different widths and are unsymmetrically located in a rectangular waveguide. The theoretical approach set out here is readily generalized to other geometries, including transverse nontouching strips and axial strips; it can also be extended to include more than two strips. Although the analysis is restricted to narrow strips, this restriction does not significantly limit the practical applicability of the results in circuit design.

The analysis reported here has direct application in the design of compact waveguide filters, in the determination of broad-band tuning circuits for strip-mounted diodes, and in the study of multidiode circuits.

II. REVIEW OF THE LITERATURE

Coupling between two antennas in an unbounded medium has been studied extensively, using two principal methods: numerical solution of the antenna integral equation to obtain the antenna current distribution [1]–[3], or derivation of a variational form for the antenna current [4]–[6].

Studies of coupling between waveguide obstacles in the same transverse plane have been confined mainly to the special case of symmetrically placed obstacles. Gruenberg [7] considered two symmetrically placed posts, such that the currents in both posts are equal in both magnitude and phase; his analysis specifically excludes the geometry for which the symmetric posts are close together. Craven and Lewin [8] analyzed a structure having three small-diameter vertical posts evenly spaced across the waveguide transverse plane; the resulting symmetry is such that the seventh mode TE_{70} is the first one to be excited, and the current in the center post may be taken to be $\sqrt{2}$ times the current in each of the two outer posts. Mariani [9] modified their results to represent each post by a T network (instead of a shunt element), with the series reactances taken to be those determined by Marcuvitz [10] for the single post, modified by an experimentally determined coupling factor.

The reactance of a general unsymmetrical multiple-strip geometry may be found by use of a singular-integral equation over a multiple interval, as shown by Lewin [11], [12] in extending the singular-integral approach which had previously been applied to symmetric double inductive apertures [13].

A recent paper by El-Sayed [14] considers a symmetrical two-post mounting structure for varactor-tuned Gunn oscillators and provides an equivalent circuit which accommodates coupling between the posts.

The analysis presented in this paper draws upon variational theory to develop an approach to two unsymmetric strips in waveguide which has the advantages of providing a value for the current-density ratio and also offering considerable physical insight into the coupling between closely spaced obstacles.

III. THEORETICAL ANALYSIS

The structure being analyzed here is shown in Fig. 1. Two infinitesimally thin, inductive, perfectly conducting strips of unequal widths w_1 and w_2 are placed unsymmetrically in a rectangular waveguide, at the plane $z = 0$. The incident dominant-mode electric field is given by

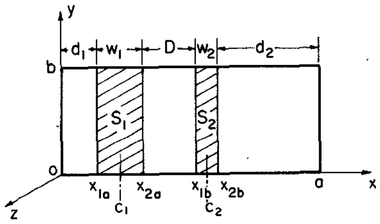

Fig. 1. Cross section of a rectangular waveguide with two infinitesimally thin strips in the same transverse plane. $C_1 = d_1 + (w_1/2)$. $C_2 = a - [d_2 + (w_2/2)]$.

Manuscript received February 14, 1975; revised July 7, 1975. This work was supported by the National Science Foundation under Grant GK-32370.

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$$E_i = \sin\left(\frac{\pi x}{a}\right) \exp(-\Gamma_1 z)\hat{y}. \quad (1)$$

For a single inductive strip in waveguide, Collin [15] shows that the normalized shunt susceptance $\bar{B}$ may be expressed in the following variational form:

$$\bar{B} = \frac{-2\left[\int_S J_y(x,y)\sin\left(\frac{\pi x}{a}\right) dx\,dy\right]^2}{\gamma_1 \sum_{n=2}^{\infty} \frac{1}{\Gamma_n}\left[\int_S J_y(x,y)\sin\left(\frac{n\pi x}{a}\right) dx\,dy\right]^2} \quad (2)$$

where

$$\Gamma_n = \left(\frac{n^2\pi^2}{a^2} - k_0^2\right)^{1/2}$$

$$\gamma_1 = -j\Gamma_1$$

and $J_y(x,y)$ is the y-directed current density in the strip of surface S.

Applying this formula to each of the strips S_1 and S_2 in turn, we obtain expressions for $\bar{B}_1$ and $\bar{B}_2$, which each represent the susceptance of one strip, if the other strip were absent (i.e., if interactive coupling effects could be neglected).

Assuming a constant current on each narrow strip, we readily obtain

$$\bar{B}_1 = \frac{-2\left[\cos\left(\frac{\pi x_{2a}}{a}\right) - \cos\left(\frac{\pi x_{1a}}{a}\right)\right]^2}{\gamma_1 \sum_{n=2}^{\infty} \frac{1}{n^2\Gamma_n}\left[\cos\frac{n\pi x_{2a}}{a} - \cos\frac{n\pi x_{1a}}{a}\right]^2} \quad (3)$$

with a similar expression for $\bar{B}_2$.

The derivation for (2) indicates that it can also be interpreted validly as giving the susceptance $\bar{B}_T$ of the total obstacle shown in Fig. 1, with S taken to be the surfaces S_1 and S_2, and $J_y(x,y)$ the current density over the two strips.

To evaluate $\bar{B}_T$ we use the following approximate form for the current density $J_y(x,y)$, suitable for use when the strips are narrow

$$J_y(x,y) = A[u(x - x_{1a}) - u(x - x_{2a})] + fA[u(x - x_{1b}) - u(x - x_{2b})] \quad (4)$$

where A is an amplitude constant and $u(x)$ is the step function defined by

$$u(x) = \begin{cases} 1 & \text{if } x \geq 0 \\ 0 & \text{if } x < 0. \end{cases} \quad (5)$$

Thus the current density is assumed constant on each narrow strip, but the current-density values on the two strips differ by the factor f, to be determined. This form is consequently an application to the two-port case of the constant-current-density form used by Collin [15].

Substituting (4) into (2) and integrating over $S = S_1 + S_2$, we obtain

$$\bar{B}_T = -\left(\frac{2}{\gamma_1}\right) \frac{(E_{1a} + fE_{1b})^2}{\sum_{n=2}^{\infty} \left(\frac{1}{n^2\Gamma_n}\right)(E_{na} + fE_{nb})^2} \quad (6)$$

where

$$E_{na} = \cos\left(\frac{n\pi x_{2a}}{a}\right) - \cos\left(\frac{n\pi x_{1a}}{a}\right), \qquad n = 1,2,\cdots,\infty$$

$$E_{nb} = \cos\left(\frac{n\pi x_{2b}}{a}\right) - \cos\left(\frac{n\pi x_{1b}}{a}\right), \qquad n = 1,2,\cdots,\infty.$$

Note that E_{na} and E_{nb} are geometric factors readily found when the structure is specified.

To evaluate $\bar{B}_T$, the value of the current-density ratio f must be determined by use of the variational principle. This f value is selected to be that which extremizes (6). Putting $(d\bar{B}_T/df) = 0$ gives an equation for f in the form

$$f = \frac{E_{1b} \sum_{n=2}^{\infty} \frac{E_{na}^2}{n^2\Gamma_n} - E_{1a} \sum_{n=2}^{\infty} \frac{E_{na}E_{nb}}{n^2\Gamma_n}}{E_{1a} \sum_{n=2}^{\infty} \frac{E_{nb}^2}{n^2\Gamma_n} - E_{1b} \sum_{n=2}^{\infty} \frac{E_{na}E_{nb}}{n^2\Gamma_n}} \quad (7)$$

from which f is readily obtained, since the infinite series are rapidly convergent because of the $(1/n^2\Gamma_n)$ term.

A current ratio F may be defined as the ratio of the total currents in the two strips. Then

$$F = \frac{fw_2}{w_1}.$$

Fig. 2 shows the ratios F and f, as a function of the width and position of the second strip, with the dimensions of the first strip held constant. It is evident from Fig. 2(a) that, when the two strips are symmetrically positioned, the wider strip has a lower current density but a greater current than the narrow strip. Fig. 2(b) confirms the expectation that, when the two strips are of equal width, the one closer to the waveguide center has a greater current density and total current. Note that $f = F$ in Fig. 2(b) since $w_1 = w_2$ in this case.

From (6), equivalent circuits may be obtained in the form shown in Fig. 3. Here

$$\bar{B}_M = \bar{B}_T - (\bar{B}_1 + \bar{B}_2) \quad (8)$$

and

$$\bar{X}_1 = (-\bar{B}_1)^{-1} \quad \bar{X}_2 = (-\bar{B}_2)^{-1}.$$

However,

$$\bar{X}_M \neq (-\bar{B}_M)^{-1}$$

and must generally be determined from (6).

Equation (6) is applicable to narrow strips of unequal width placed unsymmetrically in a transverse waveguide

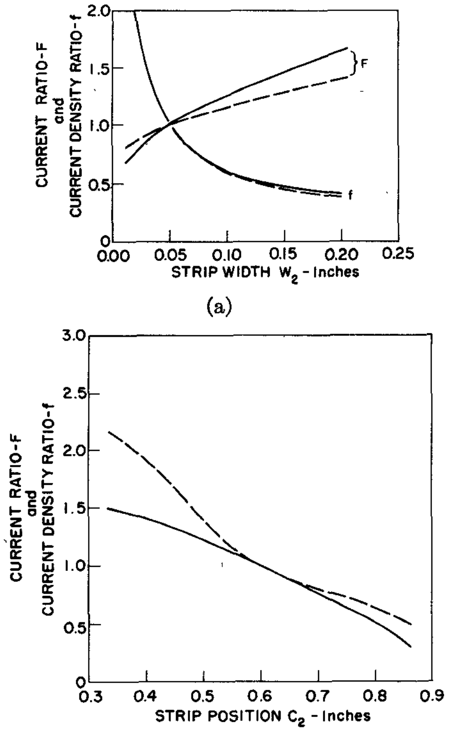

Fig. 2. Current ratio F and current density ratio f. (a) As a function of strip width w_2 for dimensions $c_1 = 0.300$ in, $c_2 = 0.600$ in, $w_1 = 0.050$ in. (b) As a function of strip position c_2 for dimensions $c_1 = 0.300$ in, $w_1 = w_2 = 0.050$ in. For both calculations $a = 0.900$ in, $b = 0.400$ in. The solid line is for frequency = 8.00 GHz, and the broken line for 12.00 GHz.

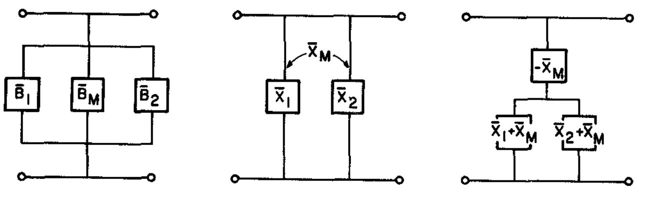

Fig. 3. Various forms of equivalent circuit for the two-strip obstacle.

plane. It is instructive to compare this formulation with that derived by El-Sayed [14] for posts of equal diameter symmetrically positioned in a transverse waveguide plane. Using Fig. 3(c) for comparison with Fig. 2(b) of [14], it is readily seen from (7) that

$$\bar{X}_M = -\frac{\gamma_1}{2}\sum_{n=2}^{\infty}\frac{1}{n^2\Gamma_n}\left(\frac{E_{na}}{E_{1a}}\right)^2 \tag{9}$$

when

$$w_1 = w_2$$

$$c_1 = a - c_2$$

and

$$E_{1a} = E_{1b}.$$

Substituting for E_{na} and E_{1a}, this $\bar{X}_M$ expression is found to be identical with the negative of the second term [14, eq. (12b)] when that term is divided by the factor k_{p1}^2 [14, following eq. (14c)]. El-Sayed identifies that term with the impedance Z_{b1} in [14, fig. 2(b)]. Thus the reactance expressed obtained here reduces to that of El-Sayed (when the gaps in his posts are short-circuited) for the special case of a symmetrical geometry.

IV. COMPARISON WITH EXPERIMENTAL MEASUREMENTS

Measurements were carried out with strips in conventional X-band waveguide having $a = 0.900$ in and $b = 0.400$ in. Results are shown in Fig. 4 as a function of frequency and distance. In Fig. 4(a) $D = 0.436$ in, while for Fig. 4(b), the strips are closely coupled, with $D = 0.101$ in. In both cases the results agree very closely with the theoretical calculation of $\bar{B}_T$. It is interesting to note that $\bar{B}_T/\lambda_g$ is almost independent of frequency, as was found by Lewin for the multiaperture obstacle [11], [12]. $\bar{B}_M$ is also shown here, and is seen to decrease slightly with increasing frequency.

Further measurements carried out to determine the susceptance $\bar{B}_T$ as a function of the distance d_1, for symmetrically placed strips, are shown in Fig. 4(c). Once again, the agreement between theory and measurement is

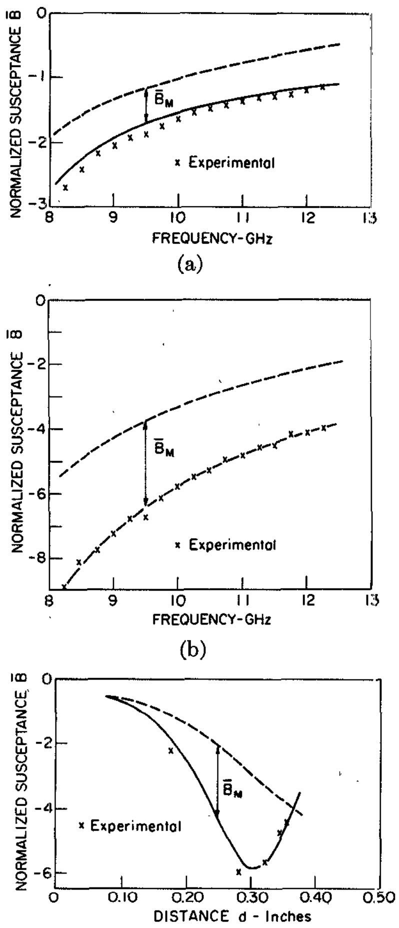

Fig. 4. Susceptance of the two-strip obstacle. The solid line shows the susceptance value $\bar{B}_T$; the broken line shows the sum $\bar{B}_1 + \bar{B}_2$. (a) $d_1 = 0.098$ in, $d_2 = 0.152$ in, $w_1 = 0.106$ in, $w_2 = 0.108$ in. (b) $d_1 = d_2 = 0.320$ in, $w_1 = 0.077$ in, $w_2 = 0.082$ in. (c) $d = d_1 = d_2$, with $w_1 = 0.077$ in, $w_2 = 0.082$ in, and frequency = 10.0 GHz. For all cases $a = 0.900$ in, $b = 0.400$ in.

close. As expected, the mutual susceptance magnitude increases when the strips are moved closer; however, for close spacing, a capacitive susceptance component decreases the overall magnitude.

Other measurements carried out for a variety of strip dimensions and positions also gave very close agreement with the theory, for narrow strips having $w/a \leq 0.15$. This restriction on strip width has not been a significant limitation upon the practical application of the two-strip obstacle in microwave circuit design.

V. COUPLING OF STRIPS IN WAVEGUIDE

Using this theory, the coupling between transverse narrow strips in waveguide can now be studied for any general value of interstrip spacing.

Although the theory is applicable to unsymmetric strips, the coupling is most readily assessed by considering two symmetrically placed transverse strips of equal width w which are moved from the waveguide sidewalls toward each other. Fig. 5 shows the calculated value of the total susceptance $\bar{B}_T$ as a function d (which is equal to d_1 and d_2 in Fig. 1). At the point T, the two strips touch to form one strip of width $2w$; at point S, the two strips overlap exactly, giving one strip of width w. Theoretical values of susceptance of a single strip of width between w and $2w$, calculated using the single-strip theory [15], are also shown on the figure, for the appropriate values of d which yield such an obstacle. In addition the curve for the sum $\bar{B}_1 + \bar{B}_2$ (which now equals $2\bar{B}_1$ for the symmetric structure) of the individual strip susceptances is shown for d values extending to point T, where the two strips touch. Experimental values are not shown in this figure since they have been shown for a similar geometry in Fig. 4(c).

It is clear from this figure that the theory developed in this paper remains applicable under tight coupling, even to the point where the strips touch and overlap. The coupling is generally inductive, but a large capacitive component arises for small interstrip spacing.

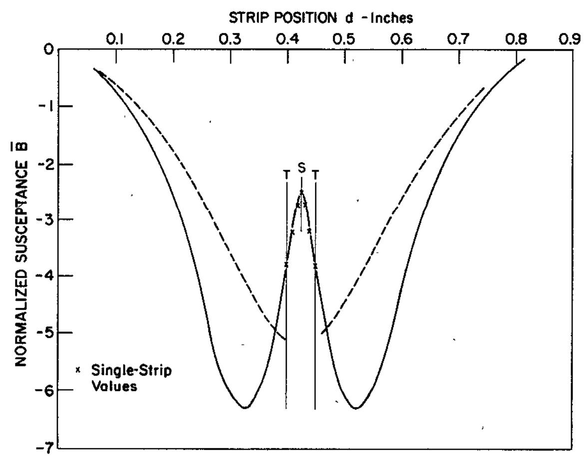

Fig. 5. Susceptance of the two-strip obstacle, as a function of the distance $d = d_1 = d_2$, compared with the single-strip susceptance when the two strips touch or overlap. The solid line shows the susceptance value; the broken line shows the sum $\bar{B}_1 + \bar{B}_2$. The curves apply to $w_1 = w_2 = 0.050$ in, $a = 0.900$ in, $b = 0.400$ in, and frequency = 8.25 GHz.

An assessment of the coupling between the waveguide strips is facilitated by a graph of $\bar{B}_M$, for the symmetric strip structure, as a function of strip separation distance D, shown in Fig. 6. It is evident that the susceptance becomes capacitive at close spacing, and that there is a maximum value of inductive susceptance coupling as D increases. It is interesting to compare this curve with that for mutual coupling between parallel dipole antennas in an unbounded medium [1].

The effect of change in the width and position of one

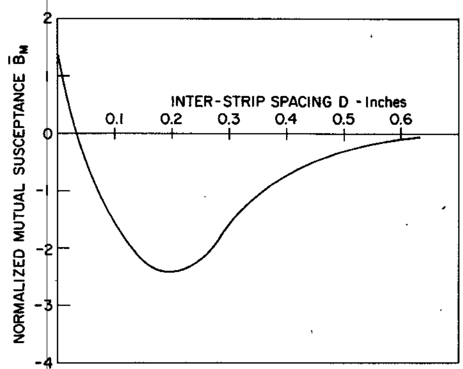

Fig. 6. Mutual susceptance $\bar{B}_M$ as a function of the interstrip separation D.

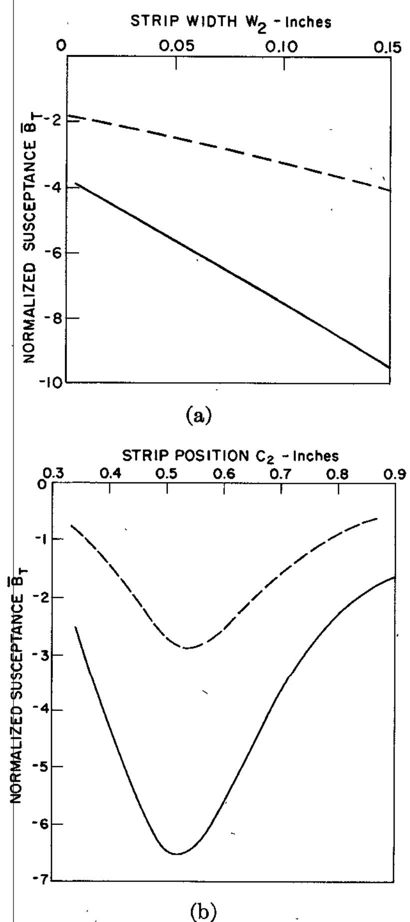

Fig. 7. Susceptance of the two-strip obstacle. (a) As a function of strip width w_2 for dimensions $c_1 = 0.300$ in, $c_2 = 0.600$ in, $w_1 = 0.050$ in. (b) As a function of strip position c_2 for dimensions $c_1 = 0.300$ in, $w_1 = w_2 = 0.050$ in. The solid line is for frequency = 8.00 GHz, and the broken line for 12.00 GHz. In both cases $a = 0.900$ in, $b = 0.400$ in.

strip on the total susceptance is shown in Fig. 7. The increase in susceptance is almost linear with width in Fig. 7(a), while Fig. 7(b) shows that maximum inductive susceptance occurs with the second strip displaced slightly from the symmetric position.

VI. CONCLUSIONS

An expression for the susceptance of two narrow transverse coupled strips has been derived using the variational technique and has been experimentally verified for a wide range of strip spacings, provided that $w/a \leq 0.15$.

The analysis presented here has application in filter design and in impedance transforming networks, in addition to the insight it provides into the coupling of proximate obstacles in waveguide.

ACKNOWLEDGMENT

The authors wish to thank a reviewer for simplifying the form of (7), and for pointing out the relationship between this work and that of El-Sayed [14].

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